$$Pe \equiv \frac{2a}{D^c} \frac{U_s}{W} \beta(1 - \phi^{5/3})$$

$$Pe_{ens} \equiv 2aU_{ens}/D^c$$

Superscripts and Subscripts

c	refers to continuous phase
d	refers to dispersed phase
α	refers to phase α ($\alpha = c$ or d)
$-$	value averaged over entire surface
ss	= steady state value
δ	= at edge of boundary layer

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Axial Dispersion of Non-Newtonian Fluids in Porous Media

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Mixing of liquids in the direction parallel to flow through porous media, usually termed axial dispersion, is a significant factor in regard to chromatography columns, packed bed reactors, and miscible displacement methods for the recovery of petroleum. For this reason axial dispersion rates have frequently been investigated, but practically all investigations have employed low viscosity Newtonian fluids such as water and light hydrocarbons. Only one publication (Wen and Yim, 1971) has reported data for axial dispersion of non-Newtonian fluids flowing in porous media, and they found their data for non-Newtonian fluids were correlated by a previous comprehensive correlation for axial dispersion of Newtonian fluids, prepared by Chung and Wen (1968). Their polymer solutions were not highly non-Newtonian, and further work was recommended with fluids exhibiting greater non-Newtonian characteristics. In this research, pseudoplastic fluids having a power law exponent as low as 0.6 were employed at very low flow rates to facilitate the observation of non-Newtonian effects on axial dispersion rates. Details of this study are reported in Payne (1971).

EXPERIMENT

The flow system used in this investigation was a vertically oriented glass bead pack, the properties of which are shown in Table 1. Glass beads of 470 micron nominal size were packed into the flow cell while vibrating the cell. To obtain a one dimensional flow situation necessary for axial dispersion measurements, liquid distributors were installed at each end of the flow cell.

Solutions of 0.5 and 0.9% Polyox WSR-301, a poly(ethylene oxide) water soluble resin manufactured by Union Carbide Corporation, were used as the non-Newtonian fluids in this study. The rheological properties of these solutions, measured with a capillary viscometer, are shown in Figure 1. Neither solution exhibited power law behavior over the range of shear rates investigated as evidenced by the curvature of the log-log plots of shear stress versus shear rate.

A 90% solution of glycerol having a viscosity of 57 cp was used as a Newtonian fluid for comparisons with the Polyox solutions.

A Dupont Pontamine Sky Blue 6BX dye at a concentration of 30 ppm in the polymer or glycerol solutions was used as the tracer in this investigation.

These studies were conducted by displacing an undyed solution from the bead pack with a dyed solution at a constant rate or visa versa. Vertical, downward flow was used in all displacements, and the entire apparatus except for the pump was thermostated in an air bath at 27°C. The effluent from the pack was collected in small samples by an automated fraction collector and then analyzed for tracer concentration with spectrophotometer. Typical breakthrough profiles for the glycerol and 0.9% Polyox solutions are shown in Figure 2.

DISCUSSIONS OF RESULTS

The major variable in the miscible displacements was the velocity of flow through the bead pack. Reynolds numbers based on the particle diameter and interstitial velocity ranged from 3×10^{-4} to 20×10^{-4} for the glycerol solution. The approximate Reynolds number range for the

Table 1. Properties of Bead Pack

Packed length	59.5 cm
Flow cell diameter	3.81 cm
Effective bead diameter	0.0355 cm (calculated) 0.0374 cm (measured)
Standard deviation of bead diameter	0.0019 cm
Void volume	248 cm^3
Porosity	0.365
Permeability	1.07×10^{-6} cm^2

0.9% Polyox was 0.9×10^{-5} to 28×10^{-5} and for the 0.5% Polyox Reynolds numbers ranged from 2×10^{-4} to 15×10^{-4}. These Reynolds numbers were based on apparent viscosities in the bead pack which were estimated from the following expression for shear rate in a bundle of capillaries (Christopher and Middleman, 1965):

$$\gamma_w = \left(\frac{3n' + 1}{4n'}\right)\frac{12\,\bar{v}_0}{(150k_\epsilon)^{1/2}} \qquad (1)$$

The one-dimensional axial dispersion model, written in dimensionless form as

$$\frac{1}{Pe}\,\frac{\partial^2\bar{C}}{\partial X^2} - \frac{\partial\bar{C}}{\partial X} = \frac{\partial\bar{C}}{\partial T} \qquad (2)$$

was applied to the data obtained from the breakthrough curves. The method of Brigham et al. (1961) of plotting on probability paper was used to determine axial Peclet numbers for each displacement. Particle Peclet numbers, obtained by multiplying the calculated values by the ratio of the particle diameter to length of column, ranged from 0.2 to 0.26 for the glycerol solution, but no particular trend was noted and the standard deviation about the mean value of 0.23 was approximately 8%. However, the Peclet numbers for both Polyox solutions were lower than the glycerol Peclet numbers and trended downward with increasing velocity. As can be seen from Figure 3, Peclet numbers for both polymer solutions appeared to be approaching those of the glycerol solution at the lowest velocities studied.

Since no published experimental data are available with Schmidt numbers of the magnitude in this study, estimated as 6×10^6 for the glycerol solution and 8×10^6 and 10^9 for the 0.5 and 0.9% Polyox solutions, direct comparisons to existing correlations are not valid. Schmidt numbers were estimated from the molecular diffusion coefficient of the dye in water (Ferrel et al., 1955) by assuming the diffusion coefficient in solution is inversely proportional to the solution viscosity (Bird et al., 1960). Miller and King (1966) and Gunn (1969, 1971) have discussed liquid dispersion in the absence of molecular diffusion, infinite Schmidt number, and both have shown that Peclet numbers should be at a constant, minimum value for this situation in laminar flow. The magnitude of the glycerol Peclet numbers found here is consistent with their discussions, and the relative insensitivity of the Peclet numbers to changing velocities suggests that it is safe to assume the Schmidt number for the glycerol solution is effectively infinite over the range of flow rates studied.

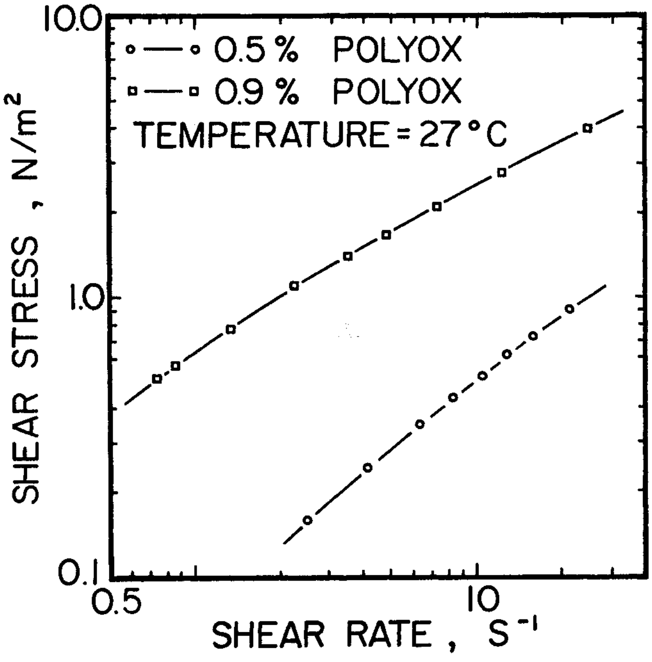

Fig. 1. Rheological behavior of polyox solutions.

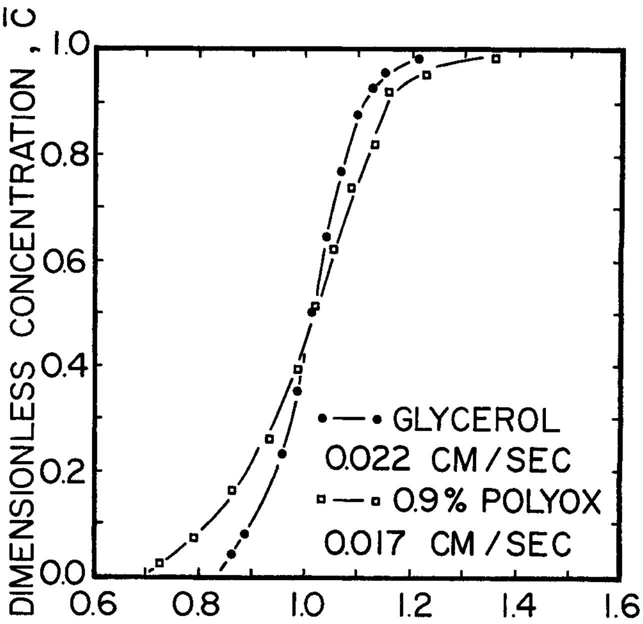

Fig. 2. Typical breakthrough profiles for glycerol and 0.9% polyox solutions.

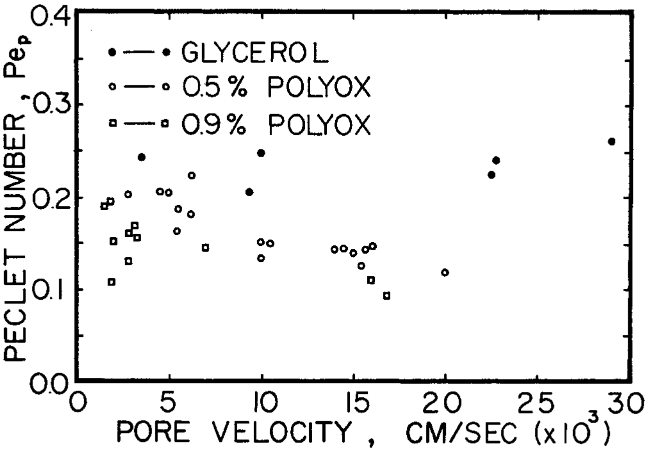

Fig. 3. Peclet numbers versus velocity for glycerol and polyox solutions.

Since the same approximate range of variables was studied for the glycerol and polymer solutions, the lower Peclet numbers and downward trend of the Peclet numbers with velocity for the Polyox solutions was attributed to the greater non-Newtonian behavior of the solutions at higher velocities. At the lowest velocities, the solutions approached Newtonian behavior and the Peclet numbers approached those of the glycerol solution.

Although the data are scattered, it is desirable to prepare a correlation of Peclet number with a parameter which represents the non-Newtonian behavior of the polymer solutions. The flow behavior index n' taken from the capillary rheogram at the shear rate obtained from Equation (1) was selected as the parameter representing non-Newtonian behavior. Peclet numbers are plotted versus n' in

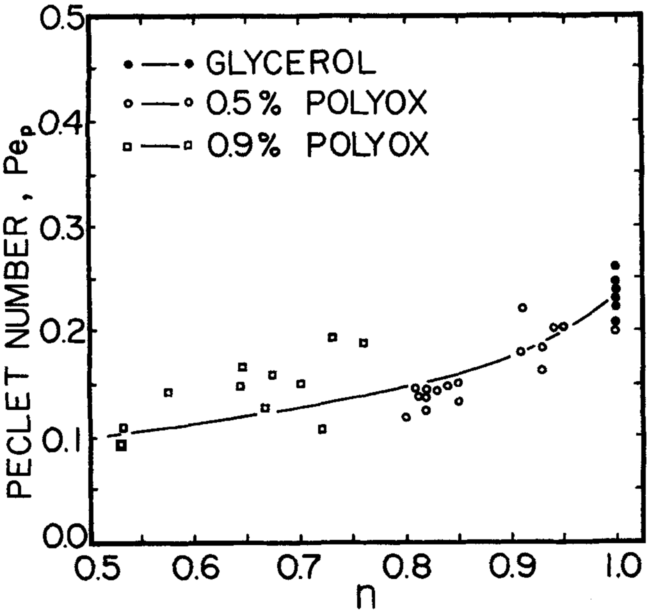

Figure 4, and a trend of decreasing Peclet number with the quantity n' is obvious. The non-Newtonian fluids reported here are not true power-law fluids as shown by the curvature on Figure 1, but a more elaborate multiparameter correlation for Peclet as a function of rheology could not be justified for the limited amount of data available.

It might be reasoned that a pseudo-plastic fluid would show either a greater or lesser amount of dispersion than a Newtonian fluid with similar properties. Flatter velocity profiles in the non-Newtonian case would contribute to less axial mixing within each channel of a porous medium, but the relative movement of fluid in adjacent channels should be greater in the non-Newtonian case because of the viscosity dependence upon shear rate. It is not possible to separate these two effects in a porous medium, but the channeling effect appears to have dominated in this study. These results should caution workers about using axial dispersion correlations prepared for Newtonian fluids for highly non-Newtonian fluids in spite of the fact that dispersion rates are independent of viscosity for Newtonian fluids in laminar flow.

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NOTATION

C = tracer concentration, kg/cm^3
C_0 = reference concentration, kg/cm^3
$\overline{C}$ = dimensionless concentration, C/C_0
D = axial dispersion coefficient, cm^2/s
d_p = diameter of glass beads, cm
k = permeability of bead pack, cm^2
L = length of a section of bead pack, cm
n' = slope of a log-log plot of τ_w versus γ_w
Pe = dimensionless axial Peclet group, $\bar{v}\,L/D$
Pe_p = dimensionless particle Peclet group, $\overline{v}\,d_p/D$
T = dimensionless time, $\overline{v}\,t/L$
t = time, s
$\overline{v}$ = interstitial velocity in bead pack, cm/s
$\bar{v}_0$ = superficial velocity in bead pack, cm/s
X = dimensionless length, Z/L
Z = space coordinate in direction of flow, cm
γ_w = shear rate at capillary wall, s^{-1}
ϵ = fractional porosity, dimensionless
τ_w = shear stress at wall of capillary, c/m^2

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